

# Defining a non-complex learning object from preschool to upper secondary school

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## ABSTRACT

The aim of this article is to analyse the aspects that teachers intend to focus on in teaching mathematics and the students' needs, i.e. what is critical for student learning. The article develops an argument for the importance of identifying the “critical aspects” as a basis for the teachers to promote student learning of Mathematics from preschool to upper secondary school. The article concludes that what teachers believe that students need to be offered concerning a specific content of Mathematics does not correspond to students' needs. Gaps between the intended and the enacted object of learning show that both the way the object of learning is offered and the way this is communicated in a teaching situation could be improved.

**Keywords:** Mathematics, aspects, critical aspects, variation theory

## 1. INTRODUCTION

This article is the first part of a longitudinal study that focuses on what happens with a learning object and students' learning in Mathematics from preschool to upper secondary school. This study started with examining what must be changed in instruction in Mathematics to improve the students' learning possibilities. Data of various kinds are collected: samples, pictures, videos and sound recordings, and documentation such as the teachers' written reports and essays. The analysis of collected data is based on variation theory. The results show there are differences between what the teacher intends to focus on and what students need to be offered to make learning happen. The results show that the teachers intend to focus on the object of learning's whole or on its separate parts, but do not focus on the relationships between those parts and how they can be related to each other in more than one way. Secondly, the

results show that the learning object should be presented in greater detail and with more variation between its aspects throughout the school system, to make the learning situation as fruitful as possible. By describing how a non-complex object of learning (an object that has a correct answer and thus can be defined in terms of right or wrong) is presented in instruction from preschool to upper secondary school, we can ascertain how aspects of an object of learning can be handled to promote learning.

## 2. THEORETICAL ASSUMPTIONS

Because of the aims of the paper, we will mainly focus on one major theoretical development, namely variation theory [1, 2], which relates the students' comprehension of a specific content to the experience of the pedagogical situation in which it is met. Runesson [3] specifies that variation theory “is not a theory of the mechanisms of learning but a theory of the relation between the object of learning and the learner” (p.406). The object of learning is broadly regarded as “the complex of different ways of experiencing the phenomenon to be learned about” [1, p.162]. The objects of learning are the ends toward which learning activities are directed and how learners understand them. Variation theory focuses on the way in which a phenomenon is made visible in a teaching context. The main idea is that in order to discern a difference, we must have experienced a variation from our previous experience. To learn means to experience, while to experience means to be aware of certain aspects in a given context and relate them to this context. Discernment of these aspects varies, and receives attention as a result of different ways individuals experience things. Moreover, simply experiencing variation, which is a decisive condition for learning, can evoke discernment of these aspects. However, not all the aspects are significant for learning. A *critical* aspect of the object of learning contributes to a particular meaning in the learner's

awareness. Only variation in the critical aspects is an essential condition for learning [e.g. 1]. To help students learn such topics, teachers must be able to understand why students may experience difficulties in discerning their critical features or aspects. In the teaching context, the teacher develops the teaching material with a perception of the content, that is, an "intended object of learning". Marton et al. [2] argue that the object of learning is defined by "its critical features, that is, the features that must be discerned in order to constitute the meaning aimed for" (p.22). A critical feature is a way of "distinguishing one way of thinking from another" (p.24). The teacher can use appropriate variations within the identified space of learning to enact the object of learning [2]. What teachers/students learn constitutes the lived object of learning.

### 3. METHOD

In this project both quantitative and qualitative methods are combined and used. The results in this article are drawn from the students' tests and discussions, and aim to identify the aspects students discern when they solve various tasks. This means that aspects are identified and described as critical or not on the basis of the student groups from preschool to high school. The teachers' essays are analyzed to determine the presumptive critical aspects for the students' learning. The collected data are analyzed from two perspectives, the intended and the lived objects of learning. The teachers' essays consist of mathematical texts and tasks that focus on one or more learning objects. The analysis focused on identifying which aspects of the content teachers want to emphasise in the teaching situation, and to what extent these aspects focus on the relationships between parts and whole. Secondly, the analysis focused on identifying which aspects of the content are critical in student learning, and to what extent these aspects focus on the relationships between parts and whole. In total, 24 teachers and 245 students have participated.

### 4. RESULTS

The analysis was done in four steps. It started with identifying what delimited objects of learning (seen as wholes) the teacher intends to offer (Step 1). For example, one teacher found it necessary to focus on "Relationship between division and multiplication" (L7). This means that the teacher intended to focus on two objects of learning (division and multiplication) and the relationships between them. Then the parts of

the object of learning are identified (Step 2). For example, one teacher (L8) writes about the negative numbers that "students do not see the difference between arithmetical operation and sign". This means that there is a difference between positive and negative numbers, and that the minus or plus signs indicate the figures' value. The next step was to analyze how the parts relate to each other, and whether it is necessary to relate the parts to each other in a different way to make learning come about (Step 3). For example, one teacher (L9) indicates that a critical aspect is the "Problems with the sorting of positive and negative numbers, namely  $3 - 5$  and  $3 - (-5)$  or when signs indicate both arithmetical operation and value". Thereafter, the analysis focused on identifying when teachers mention the way in which elements are related to the whole object of learning (Step 4). For example, one teacher (L20) identifies "Relationship between mathematical operations and variables (for example, simplify the expression  $3x + 2$  or  $2(x + 5)$ " as a critical aspect. In each step it was possible to describe the focused aspects of the object of learning in percentages between how the teacher handled them in parts, relationship between parts, the parts related to each other in different ways, the parts related to the whole and the relationship between multiple wholes.

The students' tests and discussions have focused on whether the presumptive critical aspects of the object of learning in fact were critical for their learning outcome. This means the focused learning objects were the same for teachers and students. To identify the critical aspects of the students' learning, the analysis focused on ascertaining if the students have discerned the object of learning in parts, relationship between its parts, the parts related to each other in different ways, the parts related to the whole and the relationship between multiple wholes. This analysis has been done in the same four steps used in the analysis of the data from the teachers. The results show how the learning develops from a less detailed description of the object of learning to more and more refined knowledge. The concept figures and numbers (preschool) have been included in more advanced objects of learning throughout the school system: mathematical operations with natural numbers (class 1-3), mathematical operations with natural numbers and decimal numbers (class 4-6), operations with rational numbers, algebraic expressions and formulas (class 7-9) to equations, functions, derivatives, irrational numbers and so on (upper secondary school) [4, 5]. The way aspects of the original concepts (figures and numbers) have been progressively specified and developed during the school years is presented to describe how learning in Mathematics comes about at different school levels.

The aspects of the object of learning the teachers planned to focus on were classified in six levels: (a) parts, (b) single relations between parts, (c) variation in relation between different parts, (d) relation between parts-wholes, (e) wholes or (f) relation between wholes. The results are presented in Figure 1.

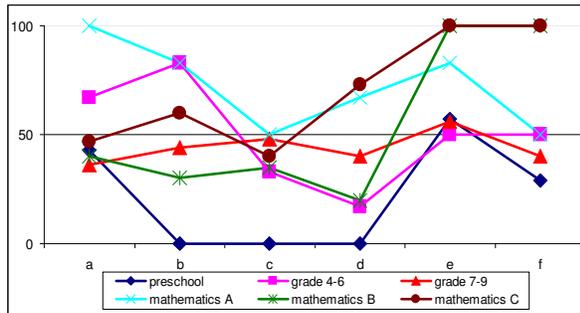


Figure 1. Classification of aspects (teachers)

Figure 1 shows how the teachers in grades 3 to 6 and in Mathematics course A intends to focus the parts (a) that constitutes the objects of learning. This phenomenon is less priority in grades 7 to 9. An example of the consequences this has for the learning outcome is Marcus (Figure 3). The question is: “Malin saves money to buy a bike that costs 525 Swedish Kronor. She has 378 Swedish Kronor. How much more does she need before she can buy the bike?” His solution (253) shows a lack of understanding between the parts and the whole. He writes:  $500 - 300 = 200$ ;  $70 - 20 = 50$  and  $8 - 3 = 5$ . As he only sees the parts, he doesn't see the importance of the order of the numbers and how these are related to a whole, and thus he writes  $70-20$ , and then he just adds the parts and gets a sum that is not correct. This is also observed in Figure 4. By focusing on different kinds of numbers (natural numbers up to 100, negative numbers or decimals), the parts should be more experienced than the whole.

In Mathematics courses B and C the teachers intend to focus on the whole (e) and the relationship between wholes (f). Examples of such complexes are equations, functions, derivatives and different relationships between these wholes.

Relationships between parts (b) are focused on mainly in grades 3-6 and Mathematics courses A and C, but this focus is not represented in preschool and only to a certain extent in grades 7-9 and in Mathematics course B. The most interesting observation is that regardless of the stage, the less focused aspect is to relate the parts to each other in different ways (c) – variation is not frequently intended or focused on in Swedish instruction in Mathematic. This area is not at all represented in

preschool. In addition, focus on the relationship part-whole (d) and the whole (e) diminishes from preschool to 9th grade. The variation of different ways to experience the parts — by themselves, together or in relation to one or more whole — aiming to gain a generalized understanding of the phenomenon, is not intended to be used or used to a high degree.

### Lived object of learning

The participating teachers conducted tests and interviews at the end of the year that focused on the selected items for learning. The test included 74 items that were analyzed and classified using the same criteria as described in the previous section. During this period, the teachers have been working with the content in the classroom as they customarily did, that is, without making any changes in their way of dealing with the content. This means that teachers' practice focused on aspects that they believed to be necessary to achieve better learning.

The results in Figure 2 show that we can conclude that the teachers thought the students in all courses had a need to understand the relationships between parts (b), that the parts can be related to each other in different ways (c) and how the parts are related to the whole (d), but this is most evident in grades 3-9 and in Mathematics course C. Moreover, we note that the teachers who taught students in Mathematics course A thought they had a need to understand the way in which various parts are related to each other (c), but as Figure 1 shows, this was not seen in their intentions when planning their lessons.

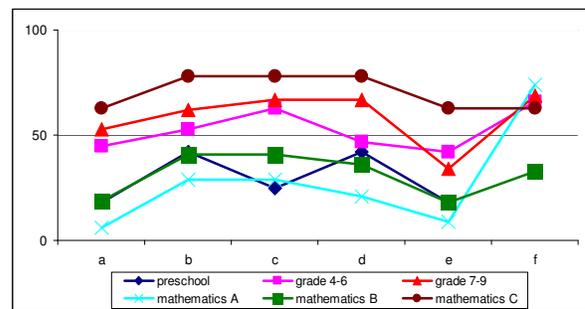


Figure 2. Critical aspects (students' perspective)

The detailed analysis of the children's work at preschool shows that they to a considerable extent discern the relationship between parts (b), relate the parts to the whole (d) and relations between wholes (f), but to a lesser extent distinguish the parts (a), and how the parts can relate to each other in different ways (c). Furthermore, children do not distinguish how different parts can be related in other ways (c).

One example is when the children were asked how many stones they had. They got seven stones, in two sequences – first five and then two more. First, they counted to five but when they saw five and two more they did not continue counting “six, seven”. Instead, 82% started over again and counted “one, two”. This indicates the children have not learned that the last numeral indicates the total number of objects.

The analysis of students' tests in grades 3-6 shows that when several mathematical methods exist in a single task, the students experience great difficulty even though the numbers that they work with are natural numbers. Regarding addition and subtraction of two natural numbers, it becomes difficult for students to identify relationships between parts (b) and relate parts to each other in a different way (c). The following example (mentioned earlier) illustrates this.

Malin saves money to buy a bike that costs 525 Swedish Kronor. She has 378 Swedish Kronor. How much more does she need before she can buy the bike?

5	0	0	-	3	0	0	=	2	0	0		
7	8	-	2	0	=	5	0					
8	-	5	=	3	=	2	5	3	k.r.			

Figure 3. Markus (student grade 4)

Markus shows that he understands the text and sees how the 378 is related to 525. In addition, Markus identifies parts (a) that constitute each number, but cannot relate those parts to each other (b) and to the whole (d). The analysis shows that in grades 7-9 the students still find it hard to identify the parts and the relationships between the parts when they work with natural numbers and multiple mathematical methods in the same task. This is seen in the following example:

$$4. \quad 21 - \frac{12}{3} + 8 \cdot 2 = 21 - 4 + 8 \cdot 2 = 25 \cdot 2 = 50$$

Figure 4. Maria (student in grade 8)

The first step in Maria's solution shows that she can identify the parts (a) and how those elements relate to each other (b), but cannot relate the new parts to each other in a different way (c). When students are working with whole numbers, there is a significant difference between how they distinguish the manner in which the parts relate to each other (b) and to the whole (d).

$$7. \quad -3 - (-4) = -7$$

Figure 5. Hanna (student in grade 8)

Hanna did not distinguish what meaning the negative signs have in this task, even though she sees that the task is about negative numbers. She believes that the negative sign, which means subtraction, together with the minus sign to mark negative numbers, yields positive characters. In addition, she believes that it is the minus sign (before 3) that is to be addressed in the response.

Solving the problems in which numbers appear in decimal form and the mathematical methods of multiplication and division occurring simultaneously is very problematic for the students. We propose that the reason for this is that these students do not distinguish between parts and how these parts relate to the whole. Consequently they do not discern how the parts relate to each other (b). The following examples illustrate this:

$$1. \quad \frac{14}{05} = 7$$

Figure 6. Johanna (student in grade 8)

In Mathematics course A, which is said to be a repetition of compulsory-school Mathematics, more critical aspects were found concerning how students discern the type of relationship between the parts that constitute the whole, how the parts can be related to each other, and to identify the whole. Furthermore, a large proportion of students show that they cannot relate two wholes to each other. Students were tested including the following tasks:

1.  $4 + 2 \cdot 3 - 7 - 5 =$
2. Solve the equation:  $x - 11 = 7$

The first task deals with a numerical expression and the other a first degree equation. There are students who solve the first task as follows:

1.  $4 + 2 \cdot 3 - 7 - 5 =$ 
  - a)  $4 + 2 = 6$   
 $6 \cdot 3 = 18$   
 $18 - 12 = 6$
  - b)  $4 + 2 = 6$   
 $3 - 7 = -4$   
 $6 - 4 - 5 = -3$

In these examples we can see that there are students who distinguish parts that constitute the numerical expression, but they cannot relate those parts to each other in different ways.

When it comes to solving a first degree equation, there are students who demonstrate that the same phenomenon remains, and even more how the consequence of this leads to mathematical aberration (see Figure 7).

Handwritten work on grid paper:

$$\begin{aligned} a) \quad x - 11 &= 7 \\ x - x &= -11 + 7 \\ x &= -4 \checkmark \end{aligned}$$

Figure 7. Karl (student studying MaA)

In Mathematics course B, the proportion of students who cannot discern the parts (a), the relationship between the parts (b), the parts relate to each other in a different way (c) increases. This increase is even higher in Mathematics course C. In this course, students have to factorize an expression of the third degree and to solve the corresponding equation of third degree. The expression was in the form:

$$a^3(a + b) - c(a + b)$$

and the students were supposed to understand that  $(a + b)$  could be factorized to get  $(a + b)(a^3 - c)$ . No student could factorize properly, which meant that the solution rate was 0%. Many others simplified the expression by creating an expression of the fourth degree. Three students felt that the term was already factored. In the second part of the task the equation

$$a^3(a + b) - c(a + b) = 0$$

was to be solved. The students who created a fourth degree equation could not do this. Since no one solved the factorization the zero product method could not be used. Thus, no student solved this task. One disadvantage of the task was to solve a third degree of expression where all roots are real.

Another task was to determine the value of  $x$  from a given expression in the limiting area  $A(x)$  that gives the least restriction area. It was not explicit in the task that it is a min/max problem, solved by the function derivative, so the students themselves should understand this. 55% of the students understood the relationship between the given function and the function's derivative.

The last example is physics data from the Physics A, where the students got the description "A stone thrown straight up, turns and falls to the ground.". Then the students had to choose which of six graphs described the stone's acceleration. If you draw an  $s$ - $t$  graph and understand that acceleration is the second derivative, i.e.  $a(t) = s''(t)$  it is possible to mathematically obtain the graph of the acceleration. There was only one graph that had constant acceleration. The solution rate was only 3%, despite the fact that the students have been taught accelerated motion in Physic A. One disadvantage of the task was perhaps that the reference direction in the graph was

such that the acceleration was positive. Normally, increased height is drawn in the positive direction of the  $s$ - $t$  graph and this would then give a negative value of acceleration.

We note that there are important differences between what teachers think students need to learn in a teaching situation and what students need to be offered in such a situation. The analysis shows that teachers from preschool to upper secondary school plan to focus on the object of learning as a whole and the parts that constitute the whole, but they take it for granted that the students should see the relations between different parts and see if the parts can be related to each other in a different way. The result shows that it is exactly these relations the students need to understand, and be guided to focus upon in the learning situation.

## 5. DISCUSSION

The results presented in this article show that what teachers believe that students need to be offered concerning a specific content of Mathematics does not correspond to students' needs. Gaps between the intended and the enacted object of learning show that both the way the object of learning is offered and the way it is communicated in a teaching situation can be improved. The analysis of the lived objects of learning shows that, in order to improve the mathematical communication, teachers may need to focus more on the parts that constitute the whole, the relationship between these parts, and to relate parts to each other in different ways to convey a general mathematical principle.

In the case of arithmetic and algebraic expressions, we note that it is the way the parts relate to each other that cause problems for students. Students seem to recognize parts and wholes, but cannot relate the parts to each other in different ways, which poses difficulties when they meet new tasks and problems.

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