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REVIEW



Low-achieving grade K-3 children's early numeracy competences: a systematic literature review

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ABSTRACT

This systematic review analyses the research results of lowachieving grade K-3 children's numeracy competencies by investigating the research approaches used, the definitions of low achievers and the numeracy competencies reported. 18 articles, identified in ERIC, PsycINFO and Web of Science, were selected for further analysis. The results show that the main part of the studies used a fixed-strategy design, mainly reporting on children's numeracy competencies at a group level in which the children's numeracy competencies were summarily described and focused on difficulties and common errors. Identification of what is defined as low achiever was based on test results from both standardised and non-standardised tests, as well as teacher assessments. The predominant numeracy competencies assessed were basic facts (automatic recall 0-20) and arithmetic skills (addition and subtraction), as well as competencies related to counting. Analyses of the children's understanding when they do not follow the typical way of learning were not found, which indicates the need for a qualitative approach to the quantitative research results in order to provide deeper understanding of children's ways of understanding and operating with numbers.

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KEYWORDS

Early numeracy; grade K-3; low achievers; school settings; systematic review

Introduction

The relationship between early numeracy competencies and later achievement in mathematics is well reported (Duncan et al. 2007; Stock, Desoete, and Roeyers 2007). In order to identify low achievers early on, practitioners need to have knowledge about the various dimensions of what it means to have weak numeracy competencies (cf. Gersten, Jordan, and Flojo 2005) and on critical aspects in early numeracy development (cf. Clements and Sarama 2014). To address the issue, this paper summarises empirical literature that reports on low-achieving children's numeracy competencies in educational settings at a K-3 level. Further, suggestions on what is needed in future research from an educational perspective are put forth. Previous reviews differ from this review in that other authors have analysed effective interventions (Kroesbergen and Van Luit 2003; Mononen et al. 2014), employed a neuropsychological approach (Raghubara and Barnes 2017) or used different inclusion criteria regarding participants (Sinnakaudan et al. 2016) (for more details see section Previous reviews). Based on a meta-analysis of six longitudinal studies, Duncan et al. (2007) found that the strongest predictors of later school achievement were school-entry mathematics skills, which were argued to be more important than reading and attention skills. Focusing on mathematics learning, Aunio and Niemivirta (2010) and Locuniak and Jordan (2008) report on a correlation between kindergartners' early number sense and arithmetical skills in early grades. Furthermore, children who experience difficulties in mathematics in kindergarten often show low mathematics skills growth through their time in elementary school (Aunola et al. 2004; Morgan, Farkas, and Wu 2009). The areas in which children with mathematical difficulties usually have significant problems is early numeracy and basic arithmetic skills (Mazzocco 2007). Correspondingly, Baroody, Bajwa, and Eiland (2009) argue that the causes of difficulties with mastering basic facts (i.e. memorising basic combinations such as 9+7=16 and 16-9=7) is due to inadequate opportunities to develop number sense (e.g. patterns, relations, algebraic rules, automatic reasoning processes, and facts).

Early numeracy

In this section, a brief overview of research about critical aspects in early numeracy development is reported. Comprehensive research reviews on mathematical learning stress that an understanding of quantities and numbers, as well as an ability to operate with numbers, are important aspects of early mathematical development (Dowker 2019; Kilpatrick, Swafford, and Findell 2001). McIntosh, Reys, and Reys (1992) use the term number sense for describing the ability to use this understanding in 'flexible ways to make mathematical judgements and to develop useful strategies for handling numbers and operations' (3), whereas Aunio et al. (2009) use early numeracy when referring to children's abilities to understand and operate with numbers. Based on longitudinal studies, Aunio and Räsänen (2016) identified that core developments intertwined with numeracy competencies in mathematical development among children aged five to eight years. These competencies were categorised into four groups: (1) symbolic and non-symbolic number sense; (2) understanding mathematical relations (early mathematical-logical principles, arithmetic principles, mathematical operational symbols, placevalue and base-ten system); (3) counting skills (knowledge of number-symbols, number word-sequence, and enumeration with concrete objects); and (4) basic skills in arithmetic (arithmetic combinations, and addition and subtraction skills with number symbols). Based on these findings, they proposed a working model for teachers of core numeracy competencies to focus on in early mathematics education. The described core competencies mainly overlap with Dowker's (2001) critical components in arithmetic development: principles and procedures related to counting, written arithmetic symbolism, place value in arithmetic, understanding and solution of word problems, translation between concrete verbal and numerical formats, use of derived fact strategies for calculation, arithmetic estimation, and number facts retrieval. The term early numeracy is used in this study as an overarching construct to define children's



abilities to understand and operate with quantities and numbers in accordance with Aunio et al. (2009).

Low achievers

In this review, focus has been on what is described as 'low achievers'. This concept has been the label for atypical knowledge development, especially regarding decreased knowledge in relation to what is defined as typical numeracy development. Mårtensson (2017, 29) addresses the issue of typical and atypical development out of a special didactics perspective by describing special didactics as the 'care, upbringing, and education of children (and adults) with needs that differ either in quality or from what is perceived as being commonplace in a culture' (author's translation). Furthermore, the importance of meeting the individual child's prerequisites is highlighted by Scherer et al. (2017); those authors argued that it is problematic to state a general conclusion on how to best support children with mathematical difficulties owing to the complex circumstances of each individual child. The complexity of mathematical difficulties is mirrored in the terminological variety used in research. The terminology employed for describing children who struggle with mathematics reflects the interpretation of the origin of those difficulties with respect to either a biological or non-biological basis (Mazzocco 2007). Mazzocco (2007) states that the terms 'disability' and 'dyscalculia' suggest a biologically based disorder, whereas the term 'mathematical difficulties' implies poor mathematics achievement in tests, regardless of the underlying causes. Bruun (2017) likewise argues that different views of the phenomenon of mathematical difficulties can be derived from different rationales of the underlying causes of those difficulties - using either an individual-categorical approach, a relational or a system-based approach. In this study, 'low achievers' is used as an overall term for defining children who are at risk of not succeeding in school mathematics, regardless of the reason.

Previous reviews

In the following, a summary of previous reviews that target the theme of low achieving children's numeracy competencies is made with the intention to describe the rationale of this paper and how it contributes to the current literature.

Kroesbergen and Van Luit (2003) conducted a meta-analysis of 58 interventional studies for elementary school for students with special needs (i.e. students who performed at a lower level and had more trouble learning maths than their peers). The selected studies used a between-subjects or within-subjects control condition in the three domains: preparatory mathematics, basic skills, and problem-solving strategies. The results showed that interventions in the domain of basic skills, which includes numeracy, produced the highest effect sizes (Kroesbergen and Van Luit 2003). Mononen et al. (2014) instead analysed studies that focused on early numeracy interventions that used random assignment or quasi-experimental design for children aged four to seven years at risk of mathematical difficulties, showing that early numeracy interventions can effectively improve the numeracy skills of children at risk of mathematical difficulties. In another review, Sinnakaudan et al. (2016) examined Malaysian grade one pupils' underperformance in solving mathematics problems. The authors concluded that the development of number sense (especially understanding part-whole number decompositions) seemed to be the major reason for their underperformance. Raghubara and Barnes (2017), in their review of longitudinal studies, describe what neurocognitive abilities underpin preschool class children's numeracy skills.

However, no study has reviewed empirical research results – whether qualitatively or quantitatively – that specifically reports on low-achieving grade K-3 children's numeracy competencies, which this systematic review addresses. Since the choice of research approaches could affect the results, it is an important aspect to investigate. Further, while research has identified the essential parts in early maths learning (i.e. numeracy) it is also of great relevance to investigate what competencies the children are assessed for and what definitions are used to identify low achievers.

Aim and research questions

The aim of this systematic review was to synthesise research results in the education of low-achieving grade K-3 children's numeracy competencies regarding methodological approaches, definitions of low achievers, and numeracy competencies reported.

A subordinate aim was to make suggestions for future research. The study aimed to address the following questions:

RQ 1: What research approaches are predominant in educational research of low-achieving grade K-3 children's numeracy competencies?

RQ 2: What definitions are used to identify low achievers?

RQ 3: How are low achievers' numeracy competencies described?

Methods

This study was guided by a systematic review approach (Eriksson Barajas, Forsberg, and Wengström 2013), as it followed the phases of selection, appraisal, and synthesis of studies that addressed the research questions. Both qualitative and quantitative studies were included, based on the assumption that diverse types of data can provide a more complete understanding of a phenomenon than either quantitative or qualitative data alone (Creswell and Creswell 2018). The analysis follows a thematic analysis approach (King 2004), which means that themes within the three areas are analysed: approaches used, how low achievers are identified, and numeracy competencies reported.

Search procedure and study selection

To identify relevant studies for the analysis, the search procedure broadly followed the guidelines from the Swedish Agency for Health Technology Assessment and Assessment of Social Services (2014). The studies were systematically searched in three databases ERIC (via EBSCOhost), PsycINFO and Web of Science. The initial search was conducted in the Educational Resources Information Centre (ERIC), supplemented with searches in PsycINFO and Web of Science. These databases were selected with the rationale that they

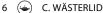
are databases which are collecting publications within the field of education. The searches were done with the assistance of librarians with expert knowledge of literary searches. The final search in ERIC was conducted on 25 October 2018, whereas a complementary search in PsycINFO and Web of Science was carried out on 13 May 2020. The three construct blocks numeracy, early education, and low achievers were identified as central due to the aim of the review. A search in the Thesaurus register in ERIC for descriptors (DE) resulted in the following: numeracy- DE 'numeracy', DE 'arithmetić, DE 'additioń, DE 'multiplication', DE 'subtraction', early education- DE 'primary education', DE 'early childhood education, DE 'preschool education, low achievers- DE 'low achievement', DE 'learning disabilities', DE 'special needs students', and DE 'underachievement'. To ensure that all relevant studies were captured, those descriptors were used as search terms together with the following synonyms for numeracy: early numeracy, number concepts, number sense, number knowledge, numerical skills; for early education: early education; and for low achievers: difficulties, struggling or at risk. The same search terms were used in all three databases. Each construct block was initially searched separately and then combined. The descriptors and synonyms for each construct were separated by OR, whereas the construct blocks were combined by AND.

The complete search string in ERIC was: (DE 'Numeracy') OR (DE 'Arithmetic') OR (DE 'Addition') OR (DE 'Multiplication') OR (DE 'Subtraction') OR numeracy skills OR arithmetic OR early numeracy OR number concepts OR number sense OR number knowledge OR numerical skills AND (DE 'Primary Education') OR (DE 'Early Childhood Education') OR (DE 'Preschool Education') OR primary education OR early education OR early childhood education AND (DE 'Low Achievement') OR (DE 'Learning Disabilities') OR (DE 'Special Needs Students') OR (DE 'Underachievement') OR low achievement OR learning disabilities OR special needs students OR underachievement OR difficulties OR struggling OR at risk. The same search terms were used in the final search in PsycINFO and Web of Science. Each search block (search terms combined with OR) resulted in thousands of results, but when combined with 'AND ' and filtered - using peer-reviewed, written in English, article papers, search terms used in the title, abstract or keywords, published between January 2007 and December 2017 a limited amount of articles remained (Figure 1). As the first part of the study was conducted in 2018, articles up to 2017 for the previous ten years were included. A complementary search was conducted in 2020 to identify if any article was missed by including two more data-based with the same search criteria as the initial one.

Inclusion and exclusion criteria

The criteria for inclusion in the review were papers with a peer-reviewed status, written in English and ones published between January 2007 and December 2017 (Table 1). The reason for limiting the search to publications between 2007 and 2017 was to search for studies in a time span representative of the current implementation of education.

The age limits for participants in the studies included a lower age limit of five years due to mathematics education usually being compulsory from that age, whereas the upper age limit of nine years was set because early numeracy competencies are taught in the first few years of schooling. This study's focus led to the search being restricted to educational studies that described low achiever's numeracy competences and the



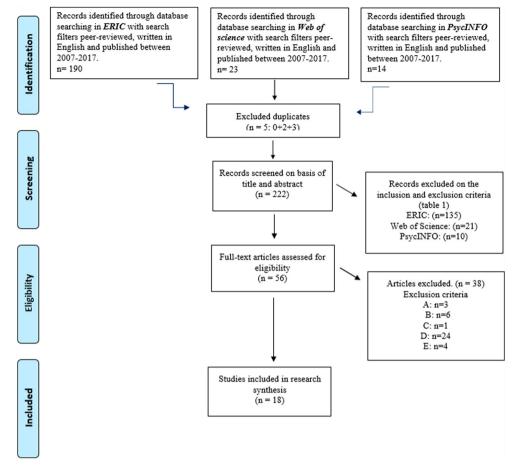


Figure 1. Flow chart of the systematic search and review process in accordance with Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA; http://www.prisma-statement.org/)

Table 1. Inclusion and exclusion criteria.

Theme	Inclusion criteria	Exclusion criteria	Exclusion category
Scientific basis	Peer-reviewed	Weak scientific basis	А
Participants	At least one group of children were classified as low achievers	No group of children were classified as low achievers.	В
	Age 5–9 years	Age younger than 5 and older than 9	
Context	Mainstream school settings	Special school or out-of-school settings, home- or clinical settings.	С
Areas of focus	Descriptions of low achievers' numeracy competences	No descriptions of low achievers' numeracy competences.	D
	,	Focus on general cognitive abilities such as working memory, language	
		Focus on test-and assessment instruments	
		and on intervention effects.	
Genre	Empirical studies	Reviews, theoretical papers	E

difficulties that low achievers' face regarding numeracy. Only studies that were conducted within mainstream school settings were therefore included: that resulted, for example, in excluding studies conducted within a summer school context or in homeor lab-based studies. Furthermore, studies that focused on cognitive or general abilities (such as working memory and language) and studies that only reported on the effects of an intervention or on changes in scores on measures on mathematics achievement, and not on describing children's numeracy competencies, were excluded. Since the intention was to find empirical studies of atypical numeracy development in mainstream school settings, the article had to define, at least one child or group of children to be low achiever(s). Regarding the genre of the articles, only empirical studies were included, theoretical articles, such as position and discussion papers, were not.

Study selection process

The study selection process appears in a flow chart (Figure 1). The filter for peerreviewed, written in English and publication years 2007-2017 resulted in 227 articles. The retrieved articles were hand-searched for duplicates and these were removed. In the next phase, the 222 relevant records were screened by title and abstract following the inclusion and exclusion criteria (Table 1), which resulted in 56 remaining articles. Those 56 articles were then assessed in full-text form for eligibility. In the eligibility phase, 38 articles were excluded based on the exclusion criteria (reasons appear in Table 1). All 18 articles were included in the synthesis.

The 18 articles were then assessed for quality using a checklist adapted from Croucher et al. (2003) (Appendix 1). All of the 18 articles met the essential quality criteria and were thus included in the study.

Data extraction

After having carefully read the full texts of the 18 articles, the characteristics that were relevant for addressing the research questions were identified. The same information was extracted from each study and systematically coded by the following: author, year, journal, aim, methodological approaches, participants, tests used, numeracy competencies assessed, and results. Furthermore, the outcomes were recorded in tabular format, which is a technique that facilitates the synthesis process across studies (Popay et al. 2006). Robson's (2011) three categories of research design (fixed, flexible or multi-strategy design) and Dowker's (2001) critical aspects of numbers were used as frameworks when reviewing the literature on methodological approaches and numeracy competencies assessed.

Synthesis

The synthesis was inspired by an integrated mixed methods research synthesis: an approach used to summarise, in words, the evidence derived from qualitative, quantitative and mixed primary studies on a common phenomenon of interest (Heyvaert, Hannes, and Onghena 2017). A narrative summary approach was employed, broadly following Popay et al.'s (2006) guidelines and involved the following: carefully reading and extracting data relevant for answering the research questions, grouping similar papers by looking for patterns within and across the groups, and narratively summarising the empirical evidence.

Results

The presented results address the research questions and correspond with the aim of this review. The analysis is based on Robson's (2011) three categories: fixed, flexible and multi-strategy design.

Research approaches

Almost all of the reviewed studies about low-achievers' numeracy competencies used a fixed-strategy design with a quantitative approach (nos. 1-3, 5-7, 9, 10, and 14-18; Table 2); only one study (no. 12) used a flexible design with a qualitative approach. The remaining four studies employed a multi-strategy design with mixed approaches (nos. 4, 8, 11, and 13). In three of these studies, the qualitative part consisted of social validation from either the teachers' views (nos. 4 and 11) or the children's views (no. 8). Furthermore, 14 of the 18 studies reported on a group level. The number of participants in the studies ranged from 3 to 660; however, more than half of the studies included over 100 participants (nos. 1-3, 5, 7, 9, 10, 13-15, and 18); and the studies were evenly distributed across grades.

Fixed-design approach

Three of the fixed design studies used an experimental design (nos. 9, 10, and 14) with either random assignment for the experimental or control condition. One study instead used a quasi-experimental design (no. 3) with no random allocation of participants to different groups, whereas two studies used a single-case multiple baseline design (nos. 16 and 17).

Among the non-experimental design studies (nos. 1, 2, 5-7, 15, and 18), where children's numeracy competencies were not manipulated, relationships between two or more variables were measured.

Flexible-design approach

Only one article adopted a qualitative approach (no. 12), using a teaching experiment design. The teaching episodes focused on the different aspects of number knowledge: conceptual, procedural, and strategic competencies. The children were organised in similar groups based on their performance on a maths test. The data collection consisted of written notations about the children's reasoning and understanding, and the analysis focused on changes in knowledge development within each group.

Multi-strategy design

Two (nos. 4 and 13) of the four studies that used a multi-strategy approach employed a quasi-experimental design with pre- and post-tests. The qualitative data in Calder Stegemann and Grünke (2014) (no. 4) consisted of audio-taped teacher meetings, teacher journals, emails, and summary notes of informal teacher meetings. In Mazzocco et al.'s



Table 2. Study, methodological approaches, number of participants, grades and math tests used.

Study number	Author	Methodological approach	Participants	Grade	Tests used for identifying low achievers (LA).
1	Aunio et al. (2009)	Fixed design Non-experimental descriptive	511	K	ENT (Finnish early numeracy test)
2	Aunio et al. (2015)		235	K	ENT (Finnish early numeracy test)
3	Bryant et al. (2008)	Fixed design Quasi-experimental	161	1	TEMI-PM test (Texas Early Mathematics Inventories)
4	Calder Stegemann and Grünke (2014)	Mixed methods design Quasi-experimental Teacher notations	37	2	Woodcock Johnson (WJ) Achivement Test.
5	Cirino et al. (2015)	Fixed design Non-experimental Explanatory	660	2	Wide Range Achievement test 3 (WRAT-3 – Arithmetic)
5	Colomer et al. (2013)	Fixed design Non-experimental Descriptive	28	1–3	AC-MT (Test di valutazione delle abilitá di calcolo)
7	Cowan and Powell (2014)	Fixed design Non-experimental Explanatory	258	2–3	WIAT-II UK (Wechsler Individual Achievement Tes)
8	Dennis, Sorrells, and Falcomata (2016)	Mixed methods design Random assignment experiment Children's self- reports	6	2	AIMSWeb (Achievement improvement monitoring system)
9	Dyson et al. (2015)	Fixed design Random assignment experiment	126	K	Number Sense screener
10	Hassinger-Das, Jordan, and Dyson (2015)	Fixed design Random assignment experiment	124	K	Number Sence screener
11	Hinton et al. (2016)	Mixed methods design Single-case experiment Teacher social validity	4	K	A school district's informal kindergarten readiness assessment
12	Lannin et al. (2013)	Flexible design Teaching experiment	16	1	Teachers identified children who were struggling in mathematics.
13.	Mazzocco et al. (2013)	Mixed design Non-experimental Qualitative error analysis	210	2–3	TEMA-2 (Test of early mathematics ability)
14	Powell and Driver (2015)		110	1	Addition fluency
15	Raddatz et al. (2017)	Fixed design Non-experimental Explanatory	127	2–3 (4)	Heidelberger Numeracy (Rechentest) Test (HRT1–4.)
16	Reynolds et al. (2016)	Fixed design Single-case experiment	3	3	Identified by their teacher because of difficulties acquiring subtraction facts.
17	Sealander et al. (2012)	Fixed design Single-case experiment	8	1–2	Precondition-to orally name and write the numerals 0–9 with 100% accuracy, more errors than corrects on A 24- item subtraction (minuends 0–9) worksheet
18	Wong, Ho, and Tang (2017)	Fixed design Non-experimental Explanatory	178	K-1	Learning and Achievement Measurement Kit 2.0.

(2013) study (no. 13), a qualitative error analysis was combined with a quantitative analysis of test scores. In another study (no. 8), a random assignment was made to one of two interventions, and the children's enhanced computational fluency was analysed quantitatively based on observations of used strategies and notations about the children's self-

reported strategies, as well as a teacher social validity questionnaire. Hinton et al. (2016) (no. 11) instead employed a single-case experiment as well as a social validity open questionnaire for the teachers to fill in after the intervention period.

In summery, in all of the fixed-design studies, the children's numeracy competences were quantitatively reported on a group level, except for the single case experiments (nos. 16 and 17) that reported the results at an individual level. In the fixed design studies, general descriptions of low achievers' performance, common errors or difficulty/difficulties were reported. In three of the mixed-design studies (nos. 4, 8, and 11), low achievers' numeracy competencies were described similarly as in the fixeddesign studies and the qualitative data were mainly used to validate the results, from a teacher's perspective. One of the mixed-design studies (no. 13) instead employed a qualitative analysis of the children's responses. Unlike the fixed-design studies, the flexibledesign study made detailed descriptions of what numeracy competencies the different performance groups developed and on what difficulties each of those groups still faced after having participated in the teaching experiment. The results were, just as in most of the fixed- and multi-strategy design studies, reported on at the group level.

Definition of low achievers

Low achievers were mainly defined based on their performance in mathematical tests (16 out of 18; Table 2). In those 16 studies, both standardised (national or regional standardised tests) and non-standardised tests (i.e. worksheets with basic facts and a kindergarten readiness test; nos. 17 and 11) with either fixed (scores) or relative (percentile or standard deviation) criteria were used. Whereas in two of the studies low achievers were defined by their teachers based on how well they performed in mathematics compared with their peers (no. 12) or if they showed difficulty in acquiring subtraction facts (no. 16).

Furthermore, the use of terminology for describing low achievers varied among the included studies as well as the cut-off criteria for defining them. Most of the studies used an achievement level at or below the 25th percentile or 1-2 SDs below the mean on standardised tests as cut-off criteria, using different terminology; 'low achievement' (nos. 2, 7, 9, 13, and 18), 'at-risk' (no. 3), 'academic difficulties' (no. 5), 'early numeracy difficulties' (no. 10) or 'moderate difficulties' (-1 to -2 SDs) (no.6). The term 'at-risk', however, was also used to identify the 30 percent lowest-performing children (no. 4).

Performance below the 11th or 10th percentile or 2 SDs below the mean on a standardised test was also used as a cut-off criterion for defining low achievers i.e. 'mathematical learning disabilities' (MLD) (nos. 7, 13, and 14), 'severe difficulties' (no. 6) or 'developmental dyscalculia' (no. 18). The term MLD was also used for individuals who scored fewer than 20 percent correct answers on a computation fluency test (no. 8).

Another term used was 'arithmetic disorder' to denote children who performed on or below a fixed score on a numeracy test (no. 15), whereas the term 'mathematical disabilities' (no. 17) was employed for defining individuals with a higher frequency of errors than correct answers on a subtraction worksheet. Still another study used 'low performers' when referring to children who had special educational needs (no. 1) while another study categorised children as having 'mathematical difficulties' based on their performance in a school district's kindergarten test (no. 11). The term 'struggling in



mathematics' (no. 12) was employed in one of the studies for children who were identified by their teacher, whereas another study used learning difficulties for children who were identified as having trouble with acquiring subtraction facts (no. 16).

Numeracy competences assessed

An overview of what numeracy competencies the children were assessed on, for identifying low achievers and competences further explored in the studies (low achievers) appears in Table 3. The analysis resulted in 22 different competencies organised in accordance with Dowker's (2001) eight themes.

The numeracy competencies that were mainly assessed in the studies (15 out of 18) were basic facts and arithmetic skills in addition and subtraction. In six of those 15 studies, the competence on how to use these skills in word or story problems was also investigated. All but one study (eight of nine) conducted in the lower grades (K-1) assessed some aspects of counting, whereas the focus in grades 2 and 3 was on basic facts and arithmetic skills (nine of nine). More than half of the included studies also assessed the understanding magnitude of numbers (nos. 3, 5, 6, 7, 9, 10, 12, 13, 15, and 18). One competence that was less frequently assessed was the ability to represent quantities and numbers in different formats (nos. 5, 7, 11, 15, and 18). In contrast, calculation strategies, estimation and mathematics only were assessed in a few of the studies.

Table 3. Numeracy competencies assessed in the studies.

Themes and core categories	Study number (Table 2)	
Principles and procedures related to counting		
Number words	1, 2	
Number recognition/identification	6, 9, 10, 11, 12, 13, 14	
Writing numbers	4	
Counting	5, 6, 9, 10, 11, 13, 14	
One-to-one correspondence	1, 2, 12, 13	
Resultative counting/cardinality	1, 2, 11, 13	
Structured counting	1, 2	
Number combinations	9, 10, 14	
Number sequences	3, 4, 5, 7, 12	
Conservation of quantities	12, 13	
Understanding of numbers/place value		
Numeral comparison	3, 5, 6, 7, 9, 10, 12, 13, 15, 18	
Place value	3, 5	
Understanding and solutions of word problems		
Understanding numbers in daily life	1, 2	
Story/word problem	4, 5, 7, 9, 10, 12, 14	
Transition between concrete, verbal, and numerical formats		
Verbal and numerical matching	15	
Quantities and numerical matching	7, 11, 15, 18	
Graphic counting, subitizing	5, 15	
Use of derived-fact strategies for calculation		
Calculation strategies	12	
Basic facts/arithmetic skills		
Addition and subtraction of numbers (mainly 0-20)	3, 4, 5, 6, 7, 8, 9, 10, 13, 14, 15, 16, 17, 18	
Arithmetic (2- and 3-digit numbers)	7	
Estimation		
Approximate quantity estimation	15, 18	
Mathematics vocabulary		
e.g. more/less than, add, equal	4, 10	

Results regarding descriptions of low achievers' numeracy competencies

Descriptions of low achieving children's numeracy competencies in the included studies are summarised in Table 4.

Competencies within the themes count, basic facts/arithmetic skills and the understanding of number values were the main themes assessed for identifying low achieving children. These were also the themes most frequently reported on (nos. 1-3, 5-7, 9, and 11-18). Some studies (no. 5, 15, 16, and 17) conducted in grades 2 and 3 described low achievers' difficulties with subtraction both regarding number facts (0-9) and calculation. The results also demonstrated that five-year-old children (Grade K) had problems with synchronous counting and enumeration (i.e. one, two, three, etc; nos. 1, 2, 11, and 12). However, the results from another kindergarten study (no. 9) showed that the children could enumerate to 20 or 30. Noteworthy in this perspective is that some children in grades 1 and 2 also made counting errors when counting to 20 or 50, respectively. The numeracy competence reported on as problematic, despite the grade, was comparing numerals such as 'which is more, 69 or 71?' (nos. 3, 6, 7, 12, 13, 15, and 18).

The results showed that the descriptions focused on describing difficulties and common errors and not on what numeracy knowledge the children demonstrated. Further, since the children's numeracy competencies were mostly quantitatively reported on a group level, the numeracy competencies reported were predominantly descriptions of the most common or typical behaviour. Only the study with a flexible design (no. 12) gave a detailed description of low achievers' competencies and showed differences in performance and understanding.

Discussion

This review made a synthesis of empirical research results that report on low-achieving grade K-3 children's numeracy competencies in mainstream school settings regarding methodological approaches, definitions of low achievers, and numeracy competencies reported. In the fixed-design studies, the children's numeracy competences were quantitatively reported in terms of composite scores. The descriptions of low achievers' numeracy competencies contributed to information on what characterise and typify low achievers, predominantly in terms of difficulties and common errors. However, detailed descriptions of differences regarding low achievers' performance, abilities and understanding were lacking. From an educator's/special educator's perspective, it may be difficult to only use a quantitative approach while reporting results when investigating low-achieving children's numeracy competencies. The quantitatively reported results do not provide insight into children's understanding and experiences. Such results mostly indicate whether a child can provide a correct answer to a specific task. Unlike the fixed-design studies, the flexible-design study made comprehensive descriptions on the different groups' various ways of understanding, how they operate with numbers, and what numeracy competencies were enhanced after participating in a teaching experiment, as well as what difficulties each of those groups still faced after having participated in a teaching experiment. This type of information allows educators to have a better understanding of the profiles of children with low achievement in mathematics, which



	Table 4.	Descriptions	of low achievers'	numeracy	competencies.
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Study no.	Descriptions of low achievers' numeracy competencies.
1	 Weak counting skills such as number words, synchronous counting, resultative counting and numbers in contextualised problems
2	• It was hard for low achievers to enumerate and by that solve problems requiring counting.
3	 Difficulties when deciding which of two numerals represent the smallest amount, or whether the quantities are equal
4	 The LA children used finger tapping when solving problems while higher functioning children used drawing, tally marks or solved the problem in their head.
5	 Struggling with the exact coding of numerosites smaller than 10, and the transcoding between those quantities and the symbolic numbers. Number- line estimation was hard.
	Weak performance on basic facts on both single-digit addition and subtraction
6	 30% of ADHD children in grade 1 made counting error when counting from 1 to 20. The results were the same in grade 2 when counting from 1 to 50. Difficulties with writing transfer to the property of the property
	 Performed well when comparing two written numbers or ordering series of numbers (for example 36, 15, 576 and 154).
7	 Single digit numeral comparison was difficult. Number system knowledge was hard such as which is more 69 or 71, what number comes five numbers after 49, number sequence backwards from 325 to 317.
	Difficulties with estimation (estimate the position on a number line on a scale from 1 to 1000)
8	 In-effective strategies such as counting on or counting all in addition and in subtraction counting all or counting down.
9	 Kindergarteners could enumerate to 20 and 30 but beyond that was hard. Weak numeral recognition beyond 10. the easiest number operations for the children to answer (with the use of objects or drawings) were word problems with the number combinations 2+1, 4+3 and 2+4, whereas 7-3 and 6-4 were more difficult
10	Many Kindergarteners did not understand the concepts 'more than' and 'less than'
11	 5-year-old children could count to 10 but not synchronously. The children did not demonstrate cardinality knowledge
12	 All children demonstrated difficulties with strategic competencies but varied in procedural fluency and conceptual understandings.
	 All children demonstrated a strong understanding of the concepts of cardinality and one-to-one correspondence.
	Some children however demonstrated difficulty with keeping track of counted items but a strong
	knowledge of the counting sequence.Some showed a lack of understanding of the order-irrelevance and conservation of quantity.
	 Most children performed better on oral magnitude comparisons than on symbolic items with the same numbers. The children correctly identified that 'fiftythree' was more than 'thirtyfive' when provided the
	 number words but not when provided with symbols. Misconception identified: 53 and 35 are the same because they both consist of threes and fives. difficulties with creating a representation for the problem situation
13	 A frequent incorrect response to the question of the largest three-digit number was 900. Some of the low achievers further responded 90 for the largest two-digit number. The children show differences in their understanding of the smallest one-digit number, some answered 1 and some 0.
14	• Difficulties with addition fluency on time (1 min, 25 items) with sums to 12.

Table 4. Continued.

Study no.	Descriptions of low achievers' numeracy competencies.
15	 It was hard to compare two one-digit Arabic numbers. Low achievers in grade 2–4 had difficulties with addition, subtraction and multiplication calculation, both on fact retrieval (1+6) and more difficult calculation tasks (231–17).
16	subtraction facts were hard for children, including grade 3
17	• Most of the children in grade 1 and 2 were unable to answer written subtraction problems in the range 0–9.
18	• Children that scored below the 10th percentile (Developmental dyscalculia DD showed difficulties with comparing non-symbolic quantities by deciding which array contains more dots. This was not the case with LA (11th to 25th percentile). Both groups showed difficulties with dot – number comparison tasks, number line estimation (0–100) and number magnitude (which of two single digits represented a larger numerical value).

may be useful when identifying and assisting low achieving children's numeracy development.

Four of the included studies reported on an individual level, however, none of these studies made detailed descriptions on low achievers' numeracy competencies, which may be needed in future research. The importance of meeting the individual child's prerequisites is validated by Scherer et al. (2017) who argued that it is problematic to state a general conclusion on how to best support children with mathematical difficulties owing to the complex circumstances of each individual child. Furthermore, according to Bruun (2017), quantitative scores on a test may be interpreted as a deficiency in the individual – separated from the environmental circumstances. Therefore, a combination of both quantitative and qualitative data could provide more robust data on how to identify and assist low achievers in their numeracy development.

There was an inconsistency in the reviewed literature on how low achieving children are defined as well as the terminology used.

The findings indicate that low achieving children were mainly defined based on low performance on operations with numbers, which is problematic due to research suggesting that the abilities to understand and operate with numbers are developmentally intertwined (Aunio and Räsänen 2016; Kilpatrick, Swafford, and Findell 2001; McIntosh, Reys, and Reys 1992). The competencies that were most frequently assessed for identifying low achievers were linked to the theme's basic facts/arithmetic skills. All of the children in grades 2 and 3 were assessed on competencies within this theme, whereas children in grade K-1 were assessed on counting competencies; both themes mainly targeted operations with numbers (Kilpatrick, Swafford, and Findell 2001). Half of the studies however also assessed children's ability to compare numbers, which requires an understanding of the value of the number.

Furthermore, the terminology used for defining low achievers in the examined studies differed, a finding that is consistent with previous research about mathematical difficulties (Mazzocco 2007). The terms 'academic difficulties', 'early numeracy difficulties', 'low achievement', and 'at-risk' were predominantly used to define children with belowaverage to low-average performance (i.e. below the 25th percentile) on tests on mathematical knowledge. In contrast, children with more severe difficulties were defined as



having 'mathematical learning disabilities' or 'developmental dyscalculia' (i.e. below the 10th-11th percentile). However, this different use of terminology could lead to misunderstandings when interpreting and communicating research results. Therefore, the need for consensus regarding terminology is needed.

However, the most prominent finding is that there are few research results in the field of low-achievers' early numeracy competencies in educational settings. This means there is a failure to provide an in-depth understanding of what difficulties struggling learners face in understanding numbers and operations with numbers, whereby both individual prerequisites and relational circumstances are considered.

Limitations and further research

Although a systematic search was conducted in the three databases ERIC, PsycINFO and Web of Science, only 18 articles met the inclusion criteria. Furthermore, 17 of the studies were found in ERIC and one article was found in PsycINFO and Web of Science, which supports the argument that articles in the field of educational research are generally found in this database. Due to the small sample, general conclusions cannot be drawn. However, the number of articles found within each search block are considerable, but when search blocks were combined, the number of results was significantly reduced.

Specific interest was in reviewing empirical literature that reports on low-achieving children's numeracy competencies in educational settings at a K-3 level, making suggestions on what is needed in future research from an educational perspective. Therefore, only studies that described low achievers' numeracy performance were included in the review, regardless of the research approaches used. However, the descriptions of low achievers were summarily described in most of the studies, especially in the studies with a quantitative approach. Analyses of children's understanding when they do not follow the typical way of learning were hard to find, which indicates a need for qualitative approaches contributing to quantitative research results by going beyond the results of the tests to provide a deeper understanding of low achievers' competencies and understanding.

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