Forces on hockey players: vectors, work, energy and angular momentum

Nina Nässén¹, Hans-Åke Nässén², Urban Eriksson³,⁴, Ann-Marie Pendrill³

¹) Erikstorpsvägen 39 F, SE 261 61 Landskrona, Sweden
²) Tulegatan 17 A, SE 871 41 Härnösand, Sweden
³) National Resource Centre for Physics Education, Lund University, Box 118, SE 221 00 Lund, Sweden
⁴) Department of Mathematics and Science Education, Kristianstad University, SE 291 88 Kristianstad, Sweden

E-mail: nina@easyskating.se, Ann-Marie.Pendrill@fysik.lu.se

Abstract. Non-traditional examples can be very inspiring for students. This paper applies classical mechanics to different ways of skating in ice hockey. Skating blades glide easily along the ice in the direction of the blade. Horizontal forces on the skates are thus essentially perpendicular to the blade. Speed skaters glide long distances on each skate before pushing off for the next stride. A hockey player running for the puck may take a number quite short steps in a short explosive rush before shifting to longer strides, where the recurring need to change direction requires additional work by the skater. This paper investigates an alternative stride, with a longer gliding phase in a circular arc, where the centripetal force provided by the ice acting on the skates, changes the direction of motion, without the need for additional energy. In addition, the conservation of angular momentum leads to increased speed as the centre of mass is shifted closer to the centre of the circular arc. Finally, we discuss an angular momentum based technique to reverse the direction of motion as fast as possible.

1. Introduction

In our efforts to find challenging, yet interesting and realistic, examples to present for physics students, we focus here on a well-known and popular sport – ice hockey – and in particular how the players are skating. Real-life examples often engage students in discussions challenging their understanding, as investigated in earlier
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Figure 1. A sequence (right-left) of screen shots (6 per second) of a skater using the unconventional slalom technique discussed in this paper

work, e.g. on swings [1], drop towers [2], trampolines [3, 4] and roller coasters [4, 5]. In this paper we present the physics of skating in two different ways and encourage the readers to use it as an example in their teaching.

The physics of skating can involve angular momentum of figure skaters, measurement of friction numbers, studies of melting of a surface layer of ice or detailed investigations of the motion of skaters (See e.g. [6, 7]).

In this paper we focus on horizontal forces in the motion of hockey players during forward motion, including consideration of work and energy. Two of us (NN and HN) have worked as hockey trainers for many years and found that an alternative technique seems to offer comparable speed while requiring less energy from the skaters than traditional hockey strides. This paper explores the physics underlying this experience, comparing traditional hockey strides and the alternative technique, shown in figure 1, where circular arcs are used to achieve a change of direction.

As the friction in the direction of motion of skating blade is very low ($\mu \approx 0.006$), skaters can accelerate only by applying forces perpendicular to a skating blade (or by pushing with the tip of a skate). A typical starting sequence is illustrated in figure 2. The skating then shifts to forward strides, as shown in Figure 3 with the resulting tracks in the ice shown in figure 4. Marino [8] found that “80% of a skating stride is spent in the single support, or gliding, phase, and 20% in the double support, or propulsion, phase”.

Recent work on hockey skating has used multiple sensors and cameras to establish 3-dimensional kinematic profiles of the skating start as well as strides during maximal skating speed (e.g. [9, 10, 11, 12]). The image processing used in these studies be seen as an elaborate extension of open source video analysis (e.g. physlets.org/tracker/) which can used in education, e.g. to determine the
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centre-of-mass motion of a hammer [13] or a ballerina in a grand jeté [14].

Figure 2. Starting sequence, with an extra step-over.

2. Force, acceleration, work and energy

Ice hockey skating is characterized by rapid starts, runs, stops and turns. The horizontal forces on the body required for these motions must be provided by the ice. The horizontal force from the ice is responsible for the forward, backward and sideways acceleration of centre of mass of the skater, but can do no work. The energy is provided by the muscles of the skaters – who are not ”rigid bodies”. E.g. extending a leg can accelerate the centre of mass and provide kinetic energy [15, 16]. However, as the horizontal forces on a skate are essentially orthogonal to the motion of the skate itself (unless the skate slides, as in rapid braking), the changed motion of the centre of mass can be in the direction of motion of the other skate, or toward the centre of the circular arc. In this way, an increase in kinetic energy can be obtained. Skating over a distance also requires repeated changes of the direction of motion. These changes can be obtained in different ways, as discussed below.

3. Traditional hockey strides

Consider first the traditional hockey skating, as described e.g. by Stamm [17]. The acceleration in the direction of motion, increasing the kinetic energy of the body, is provided by the work done by the extending leg as seen in figure 3.

This type of motion has been analysed in considerable detail for elite hockey players, (see e.g. [9, 10, 11, 12]) where multiple sensors, as well as markers on the body and video analysis, were used to study the three dimensional motion
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Figure 3. Example of a traditional forward stride, with the right skate pushing off perpendicularly to the blade, or with the tip of the skate, followed by the left skate pushing off. The body also rotates slightly, and the arm motion from side to side reduces the sideways motion of the core of the body.

of skaters on ice, comparing the different kinematic profiles for the initial strides of the "acceleration phase" as well as for later strides. Budarick and coworkers [11, 12] found that the during the "acceleration phase, where athletes push off from a relatively fixed location on the ice, the rotational velocity is a greater contributor to forward velocity". The later strides were characterized by "a gliding push-off, in which the position of the front of the skate moves along the ice, and the extension velocity of the leg plays a larger role in the generation of forward velocity".

From the tracks in figure 3, we see that some of the acceleration obtained by extending the leg is needed to change the direction of motion. This requires energy from the skater, as discussed below.

3.1. Work and energy in traditional forward strides

In a first simplified analysis of the energy transformations during forward strides, we consider motion along the straight-line tracks at an angle $\theta$ from the average direction of motion, along the $y$ axis, as shown in figure 4. The speed at the beginning of each stride is denoted by $v_i$. 
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Figure 4. Tracks in the ice for conventional fast strides at an angle \( \theta \) to the main direction of motion, together with a definition of the coordinate system used for the analysis. The horizontal force, \( \mathbf{F} \), from the ice on the skater is orthogonal to the skating blade.

The velocity at the beginning of a stride to the right, after the push-off with the left leg, could then be written as \( \mathbf{v}_{i,r} = v_i(\sin \theta, \cos \theta) \). Before the leg pushes off for the next stride, the speed has dropped to \( \alpha v_i \), where the fraction \( \alpha < 1 \) depends on air resistance and on friction between the ice and the gliding skate. At the end of the stride, the velocity can be written as \( \mathbf{v}_{f,r} = \alpha v_i(\sin \theta, \cos \theta) \). The kinetic energy has then dropped from \( \frac{mv_i^2}{2} \) to \( \frac{m\alpha^2 v_i^2}{2} \), where \( m \) is the mass of the skater.

The kinetic energy lost during a stride, \( \Delta E_{k,0} = (1 - \alpha^2)\frac{mv_i^2}{2} \), needs to be supplied during the push-off, to allow the next stride to start with a velocity \( \mathbf{v}_{i,l} = v_i(-\sin \theta, \cos \theta) \), slightly to the left. However, from the system of the skater, moving along the track to the right, we could consider work done by the extending right leg to achieve the new velocity. (Note that changes in kinetic energy depend on the initial velocity within the reference system used [15, 16].) This velocity change can be written as

\[
\Delta \mathbf{v} = \mathbf{v}_{i,l} - \alpha \mathbf{v}_{i,r} = v_i((-1 - \alpha) \sin \theta, (1 - \alpha) \cos \theta).
\]

The skater needs to exert work \( W \), corresponding to the kinetic energy of this relative velocity, \( \mathbf{W} = \frac{m(\Delta \mathbf{v})^2}{2} \), giving

\[
W = \frac{mv_i^2}{2} \left( (1 + \alpha)^2 \sin^2 \theta + (1 - \alpha)^2 \cos^2 \theta \right).
\]
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Using $\sin^2 \theta + \cos^2 \theta = 1$, this can be rewritten as

$$ W = \frac{mv_i^2}{2} [1 + \alpha^2 + 2\alpha (\sin^2 \theta - \cos^2 \theta)] = \frac{mv_i^2}{2} (1 + \alpha^2 - 2\alpha \cos 2\theta) $$

Inserting the expression $\Delta E_{k,0} = (1 - \alpha^2)mv_i^2/2$ for the kinetic energy loss in the skating rink system gives

$$ W = \Delta E_{k,0} + (\alpha^2 - \alpha \cos 2\theta) mv_i^2. $$

The work required for the new stride is thus found to be larger than the kinetic energy lost during the previous stride if $\cos 2\theta \leq \alpha$. For the unrealistic case of no friction, $\alpha = 1$, there is no need for alternating strides and the motion can continue in the same direction with $\theta = 0$. If instead all kinetic energy were lost before starting the next stride, i.e. $\alpha = 0$, the skater is already at rest in the skating rink system and thus $W = \Delta E_{k,0}$. For $\theta = 45^\circ$, where the new stride is orthogonal to the previous stride, $\cos 2\theta = 0$ and the last term vanishes. The work required is then $(1 + \alpha^2)mv_i^2/2$, which is the sum of the work required to stop the motion in the initial direction (as seen from the skating rink) and the work to reach the full velocity in the orthogonal new direction.

This model is clearly oversimplified: The shift of the direction of the centre-of-mass motion is less than in the model, above. As the leg extends to the right, it pushes the body to the left, and the body is no longer directly over the track. The sideways motion of the arms swinging from side to side, as in figure 3, also reduces the sideways motion of the core of the body.

Hayward-Ellis et al [18] analysed the “ground reaction forces” using different arm swing techniques and found that the sideways reaction forces caused by this arm swing is comparable to the sideways force from the skates. They also concluded that the side-to-side arm swing was more effective for skating than the back-to-front arm swing used by runners, as well as by many skaters.

It can also be noted that the track in figure 3 deviates from a straight line - during the last part of the stride, the skate turns slightly outwards to allow for the push to be better aligned with the gliding on the other leg, since the horizontal part of the force from the ice, is essentially perpendicular to the skating blade.

In the next section we discuss angular momentum skating, where the change in direction is instead provided by moving along a circular arc, and the full energy provided by the work exerted orthogonally to the track is converted into kinetic energy.
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4. Angular momentum skating

In this section we discuss angular momentum skating, where the change in direction is instead provided by moving along a circular arc. In this alternative technique, the change of direction requires no additional energy from the skater. Instead, the sideways force from the ice on a leaning skate is used to creating an arc where the direction changes continuously, without requiring work by the skater. (Energy losses due to friction in the direction of motion occur, of course, independent of the technique used.)

In traditional hockey skating, after one skate, S1, leaves the ice and the other skate, S2, glides on the ice, the skate S1 is moved closer to skate S2, into a position suitable for starting the next gliding phase, at an angle to skate S1. Skate S2 then pushes off while the leg extends before skate S2 leaves the ice while S1 continues to glide, as shown in the example in figure 3.

In angular momentum skating, after skate S1 leaves the ice, instead of getting ready for the next glide while moving closer to the gliding skate S2, skate S1 is
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placed on the ice essentially \textit{parallel} to skate S2 and both skates then glide together in a circular arc. The ice provides the centripetal force, which may be distributed on both skates.

The body also leans towards the centre of the circle, enabling the total force from the ice on each leg to be along the direction of the leg. Skate S2 pushes the body towards skate S1, shifting the centre of mass closer to the centre of the circle as the leg expands. Since the force from the ice is purely orthogonal to the skate, it points to the centre of the circle and angular momentum is conserved. Moving closer to the centre then implies increased speed, as discussed in more detail in section 4.2.

Table 4) shows the different phases of a sequence of moves, connecting to the annotated sequence of screen shots in figures 6 and 7 and to the annotated track in figure 8,

\begin{table}[h]
\begin{tabular}{|l|}
\hline
a) Skate S1 leaves the ice. \\
b) Skate S1 moves close to the gliding skate, S2. \\
c) Skate S1 is placed on the ice essentially \textit{parallel} to skate S2. \\
d) Both skates then glide together in a circular arc while skates and body lean into the circle. \\
e) Skate S2 pushes the body towards the centre of the circular arc. \\
f) Final extension of the leg, shifting the load to skate S1 before skate S2 leaves the ice. \\
(The arm motion contributes to the shift of the centre of mass.) \\
\hline
\end{tabular}
\end{table}

4.1. Skating with circular arcs

The top sequence in figure 1 shows the left leg pushing the body to the centre of an arc to the right on the inside edge. The left leg pushes off and extends while
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the right skate moves in a straight line, forward and away from the left foot. The snowy part of the track shows where the sideways push has been strongest. During this part of the motion, work can be done by the left leg, to increase the speed of the skater, by shifting the centre of mass towards the centre of the circle, as discussed in more detail in section 4.2. The left leg is then pulled in and the whole body rotates to the left as the right foot changes to skating on the inside edge. No additional energy is needed for the change of direction. As the left skate makes contact with the ice again on the outside edge, it helps pushing the body to the left.

This second row of the sequence in figure 1 shows the left skate getting contact with the ice again, first on the outside edge possibly providing supporting to the right skate as the body is pushed to the left, shifting the centre of mass. After the right skate leaves the ice, the body rotates to an inside edge of the left skate, which then pushes the body towards the centre of the circular arc, which is shifted to the right.

The third row in figure 1 shows the right skate making contact with the ice, and the left skate pushing it closer to the centre of the arc. Note also that during the part when the motion changes direction, the arms are kept closer to the body, reducing the moment of inertia of the body.

Figures 6 shows an annotated selection of screen shots from figure 1, illustrating the different parts of the motion. Figure 7 shows annotated screen shots at 0.1s intervals of the motion viewed from behind. The slope of the skate indicates the direction of the force from the ice. The images also show a relatively long time with both skates in contact with the ice, as the center of mass is shifted from right to left, while the right skate pushes away.

Figure 8 shows coloured tracks for a similar move at a different part of the ice. The letters refer to the photo sequence in figure 7 and in table 1, giving an attempted match between the tracks on the ice and the position of the two skates during different parts of the motion.

4.2. Angular momentum in forward skating

A skater moving on an inside edge in a circular arc can push the centre of mass of the body towards the centre of the circle. A skater with speed \( v \) moving in a circular motion with radius \( r \) is accelerating with \( v^2/r \) (the centripetal acceleration) towards the centre of the circle. The ice must then push on the skate(s) with a force \( F_c = mv^2/r \). If a skater maintains the radius, a change of direction will be
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Figure 7. Screen shots (at 0.1s intervals) of the motion viewed from behind. The slope of the skate indicates the direction of the force from the ice. The letters correspond to the description in section 4 of the different parts of a stride.

Figure 8. Tracks on the ice with letters marking approximate locations of the two skates during of the different parts of a slalom stride defined in section 4.

achieved without providing additional energy.

Since the external force on the skater is perpendicular to the skating track, pointing towards the centre of a circle, angular momentum, \( L \), is conserved. In this case we can write \( L = mrv \), where \( r \) is the radius of the circle and \( v \) is the speed of the centre of mass. If a skater with an initial velocity \( v_0 \) entered the circular arc with radius \( R \), and then reduces the centre of mass radius to \( r \), the speed will increase to \( v(r) = v_0 R/r \). The shift of the centre of mass can be achieved by expanding the legs and/or leaning more into the centre. The force from the ice can be exerted on one or both skates. Figure 9 shows an example of the motion of the centre of mass, with constant angular momentum but shrinking radius.
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The work by the skater can be calculated by inserting the radius dependence of the centripetal force: $F_c(r) = -mv^2/r = -m(v_0R/r)^2(1/r) = -mv_0^2R^2r^{-3}$ into the expression for the work, $dW = F \cdot ds$, giving

$$W = \int_R^r F_c \cdot ds = -mv_0^2R^2 \int_R^r \frac{1}{x^3} dx = -mv_0^2R^2 \left[ \frac{-1}{2x^2} \right]_R^r$$

Evaluating the integral gives

$$W = \frac{mv_0^2}{2} R^2 \left( \frac{1}{r^2} - \frac{1}{R^2} \right).$$

This expression is identical to the change in kinetic energy related to the increased speed for smaller radii. Thus all the work done by the body to shift the centre of mass closer to the centre of the circle is converted to kinetic energy.

For the analysis in this section, it makes no difference whether the force to shift the centre of mass comes from one skate or the other, or a combination, as the mass shifts from one skate to the other, in preparation for the next stride. Leaning in towards the centre of the arc, as well as moving the arms from one side to the other also contribute to the shift of the centre of mass relative to the skating track. It can be noted that this type of propulsion can not be practiced on a skating treadmill: whereas linear motion is relative and equivalent to rest, with a moving surrounding, this does not hold for rotations and circular motions.

Angular momentum is more commonly associated with figure skating, where the body of the skater forms the centre of the circle, and the arms are pulled in to reduce the moment of inertia, leading to higher angular velocities. In this work, we have shown how angular momentum can play a role also for forward skating.

5. Reversing the direction of motion

A common challenge in ice hockey is the "tight turn", where players aim to reverse the direction of motion as fast as possible. This involves a large acceleration, and thus a large force, in a direction opposing the original motion. Traditionally, players slow down, come to a stop and then start again, using a cross-over to gain speed in the new direction as studied e.g. in [19] for 90° turns. As the Apollo 13 astronauts exclaimed "Houston, we have a problem", they discovered that they had to continue to the Moon to reverse the motion - the rocket fuel available would not be sufficient for a reversal in "free space", whereas the motion around the moon provided a centripetal force, capable of reversing the motion, without using any fuel for the operation. Again, using a narrow circular arc can offer a fast...
and energy-efficient hockey turn, described by [20] as an "open Mohawk turn". A smooth change from forward to backward skating allows the player to be able to still keep an eye on the puck. This move is demonstrated in the accompanying video abstract.

6. Discussion

This paper considers how the fundamental physical concepts of classical mechanics can be applied to an understanding of fast forward skating and a comparison between traditional hockey skating to an unconventional technique based on conservation of angular-momentum.

One of the authors (NN) has experienced the advantage of the unconventional...
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angular-momentum based technique several times during hockey training warm-up: When participants were asked to skate back and forth across the ice, she discovered that one by one, elite players using the traditional forward skating took a rest in a corner while she continued to skate, comparatively effortless. The theoretical analysis, using classical physical concepts of work, energy and angular momentum, supports her experience that the alternative technique involves less energy losses and requires less work by the skater. A more detailed analysis using multiple sensors and video analysis could be rewarding student projects for hockey-playing students.

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